Name: Solutions
Date: 2/25/2025

Math 130
Exam 1

Please show ALL your work on the problems below. No more than 1 point will be given to problems if you only provide the correct answer and insufficient work.

1. (21 points) In order to determine the percentage of high school seniors who smoke marijuana, 384 high school seniors were randomly selected and asked to honestly report if they smoke marijuana. Of those polled, 81 of them reported that they do smoke marijuana.

a) What is the population?

All high school seniors

b) What is the sample?

The 384 polled high school seniors

c) What is the population data?

The collection of yes's and no's from all high school seniors.
Yes = They smoke marijuana
No = They don't smoke marijuana

d) What is the sample data?

The collection of yes's and no's from the 384 polled high school seniors.
Yes = They smoke marijuana
No = They don't smoke marijuana

e) What is the population parameter? (give the symbol and describe it in words)

p = The percentage of all high school seniors that smoke marijuana

f) What is the sample statistic? (give the symbol, describe it in words and give its value)

$\hat{p}$ = The percentage of the 384 polled high school seniors who smoke marijuana

$= \frac{81}{384} = 21.09\ \%$

g) What is your best estimate of the population parameter?

$p \approx \hat{p} = 21.09\ \%$

2. (16 points) Data: 8 19 8 1 19 48 14 19 7 24 8 7

Data in increasing order: 1 7 7 8 8 8 14 19 19 19 24 48

For this data, find the

a) mean

$\bar{x} = \frac{1+7+\cdots+48}{12} = \frac{182}{12} = 15.17$

b) median

$\tilde{x} = \frac{8+14}{2} = 11$

c) mode

mode = 8, 19

d) midrange

$= \frac{\text{lowest} + \text{highest}}{2} = \frac{1+48}{2} = 24.5$

e) range

$= \text{highest} - \text{lowest} = 48 - 1 = 47$

f) standard deviation

$\Sigma x^2 = 1^2 + 7^2 + \cdots + 48^2 = 4450$

$\Sigma x = 1 + 7 + \cdots + 48 = 182$

$s = \sqrt{\frac{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}{n-1}} = \sqrt{\frac{4450 - \frac{(182)^2}{12}}{12-1}}$

$s = 12.39379121$

g) variance

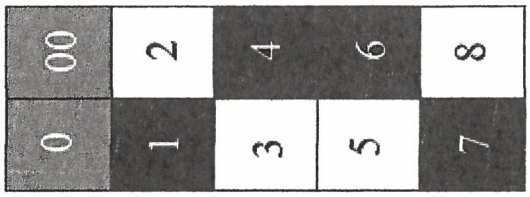
$s^2 = (12.39379121)^2$

$s^2 = 153.6060606$

3. (2 points) Suppose you have 2 data sets. The standard deviation for data set 1 is $s_1 = 7.9$ and the standard deviation for data set 2 is $s_2 = 4.7$. What does this tell you about the data sets?

Data set 1 is more spread out than data set 2 because s_1 is larger than s_2.

4. (39 points) Imagine a modified roulette wheel that only has 10 equal sized slots. To play this game, a ball spins around the wheel and lands in one of the slots. Of the 10 slots, 8 are labeled with numbers 1 through 8 and the other slots are labeled 0 amd 00. Of the numbered slots (1 through 8), half are black and half are white. The 0 and 00 slots are gray.



$B = \{1,4,6,7\}$
$G = \{0,00\}$
$E = \{2,4,6,8\}$

Let

B be the event that the ball lands in a black slot
G be the event that the ball lands in a gray slot
E be the event that the ball lands in an even slot

Find

a) S
$= \{0,00,1,2,3,4,5,6,7,8\}$

b) E
$= \{2,4,6,8\}$

c) $\overline{G}$
$= \{1,2,3,4,5,6,7,8\}$

d) $B \cup E$
$= \{1,2,4,6,7,8\}$

e) $B \cap E$
$= \{4,6\}$

f) $P(S)$
$= \boxed{1}$

g) $P(E)$
$= \frac{|E|}{|S|} = \boxed{\frac{4}{10}}$

h) $P(\overline{G})$
$= 1 - P(G) = 1 - \frac{|G|}{|S|}$
$= 1 - \frac{2}{10} = \boxed{\frac{8}{10}}$

i) $P(B \cup E)$ (not disjoint) $= P(B) + P(E) - P(B \cap E)$
$= \frac{|B|}{|S|} + \frac{|E|}{|S|} - \frac{|B \cap E|}{|S|} = \frac{4}{10} + \frac{4}{10} - \frac{2}{10} = \boxed{\frac{6}{10}}$

j) $P(B \cap E)$ (indep, ?)
$= P(B) \cdot P(E|B)$
$= \frac{4}{10} \cdot \frac{2}{4} = \boxed{\frac{8}{40}}$

k) $P(E \mid B)$
$= \boxed{\frac{2}{4}}$

l) Are the events B and E disjoint? Why or why not?

No, bec 4 and 6 are in both events.

m) Are the events B and E independent? Why or why not?

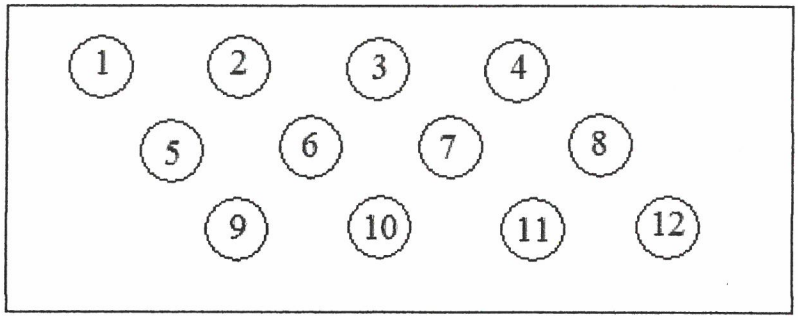
$P(B|E) \stackrel{?}{=} P(B)$

$\frac{2}{4} \stackrel{?}{=} \frac{4}{10}$

$0.5 \stackrel{?}{=} 0.4$ X

So B and E are not independent.

5. (16 points) A container contains 12 balls labeled 1 through 12 as shown below.



a) If you draw 2 balls from the container with replacement, what is the probability that both have numbers on them that are bigger than 8?

Let B be the event that the selected number is bigger than 8.

indep. ✓

$$P(B_1 \cap B_2) = P(B_1) \cdot P(B_2) = \frac{4}{12} \cdot \frac{4}{12}$$

$$= \boxed{\frac{16}{144} = \frac{1}{9} = 11.11\ \%}$$

b) If you draw 2 balls from the container without replacement, what is the probability that both have numbers on them that are bigger than 8?

indep. ?

$$P(B_1 \cap B_2) = P(B_1) \cdot P(B_2 \mid B_1) = \frac{4}{12} \cdot \frac{3}{11} = \boxed{\frac{12}{132} = \frac{1}{11} = 9.09\ \%}$$

c) If you draw 2 balls from the container without replacement, what is the probability that neither have numbers on them that are bigger than 8?

indep. ?

$$P(\overline{B_1} \cap \overline{B_2}) = P(\overline{B_1}) \cdot P(\overline{B_2} \mid \overline{B_1}) = \frac{8}{12} \cdot \frac{7}{11} = \boxed{\frac{56}{132} = \frac{14}{33} = 42.42\ \%}$$

d) If you draw 2 balls from the container without replacement, what is the probability that at least one has a number on it that is bigger than 8?

$$P(\text{at least one is bigger than 8}) = 1 - P(\text{none are bigger than 8}) = 1 - P(\overline{B_1} \cap \overline{B_2})$$

$$= 1 - \frac{56}{132} = \boxed{\frac{76}{132} = \frac{19}{33} = 57.58\ \%}$$

6. (12 points) At Lucy's taco stand you build your own tacos by choosing the type of shell, choosing the type of meat, choosing whether or not you want cheese, and choosing one condiment. The options are listed below.

Shell: Hard shell, Tortilla
Meat: Beef, Chicken, Steak, or Fish (beef and steak are red meats)
Cheese: Cheddar, No cheese (think of this as a 2nd option)
Condiment: Sour Cream, Guacamole, Pico De Gallo

a) How many different tacos can be made at Lucy's taco stand?

$|S| = \underset{\text{shell}}{2} \cdot \underset{\text{Meat}}{4} \cdot \underset{\text{cheese}}{2} \cdot \underset{\text{Condiment}}{3} =$ 48 different tacos in total

b) How many different tacos can be made that contain red meat and no cheese?

Let R be the event that the selected taco has red meat and no cheese.

$|R| = \underset{\text{shell}}{2} \cdot \underset{\text{Meat}}{2} \cdot \underset{\text{cheese}}{1} \cdot \underset{\text{condiment}}{3} =$ 12 tacos that have red meat and no cheese.

c) If Lucy randomly selects ingredients and makes a taco for you, what is the probability that it will contain red meat and no cheese?

$P(R) = \frac{|R|}{|S|} = \frac{12}{48}$

7. (3, 5, 3, 3 points) Consider the experiment where you play a single game of roulette. Let R be the event that the ball lands on red.

a) Find $P(R)$ (write this answer as a percentage)

$= \frac{|R|}{|S|} = \frac{18}{38} =$ 47.37 %

b) What does your answer in part (a) mean?

If you play a single game of roulette many times, the ball will land in a red slot about 47.37 % of the time.

c) If you were to repeat this experiment a total of 190,000 times, how many times will the ball land on red?

$\left(\frac{18}{38}\right)(190{,}000) = 90{,}000$

About 90,000 times

d) If you were to repeat this experiment infinitely many times, what percentage of the time will the ball land on red?

Exactly 47.37 % of the time.

Some formulas you may need:

$$\bar{x} = \frac{\sum x}{n} \qquad s = \sqrt{\frac{\sum (x-\bar{x})^2}{n-1}} = \sqrt{\frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}}$$

$$P(A \cup B) = P(A) + P(B) \qquad P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A) \cdot P(B) \qquad P(A \cap B) = P(A) \cdot P(B \mid A)$$

$$P(\overline{A}) = 1 - P(A) \qquad P(at \ \ least \ \ one) = 1 - P(none)$$